Statistics

Lecture 8

Feb 19-8:47 AM

Given $P(E) = .04$

SG 12

1) Find $P(\overline{E})$

$P(\overline{E}) = 1 - P(E) = 1 - .04 = .96$

2) Find odds in favor of event E.

$P(E) : P(\overline{E})$

$.04 : .96$ → using Calc to simplify

$.04 \div .96$ MATH 1: ▶Frac Enter → $\frac{1}{24}$ → 1 : 24

3) Find odds against event E.

Reverse the odds in favor

24 : 1

Jul 9-4:30 PM

Suppose odds in favor of event E are $3 : 37$

1) odds against Event E.

$37 : 3$

2) $P(E)$ in decimal.

$P(E) = \frac{3}{3+37} = \frac{3}{40} = .075$

3) $P(\bar{E})$ in % notation.

$P(\bar{E}) = 1 - P(E) = 1 - .075 = .925$

$= 92.5\%$

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A deck of cards has 50 cards, 24 are red, 8 face cards, and 3 aces.

What are the odds in favor of selecting

1) Red Card

Red : # $\overline{\text{Red}}$

24 : 26

Divide by 2

12 : 13

2) Face Card

Face : # $\overline{\text{Face}}$

8 : 42

Divide by 2

4 : 21

3) an Ace

Ace : # $\overline{\text{Ace}}$

3 : 47

4) Face or ace.

Face or Ace : # $\overline{\text{Face or ace}}$

11 : 39

Jul 9-4:40 PM

Multiplication Rule

Keyword AND

Multiple Action Event

Case I: If A & B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$

Ex: Suppose $P(A) = .3$, $P(B) = .7$, A & B are independent events

$P(A \text{ and } B) = P(A) \cdot P(B) = (.3)(.7) = \boxed{.21}$

A happens, then B happens

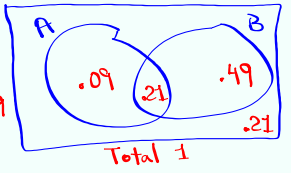
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .3 + .7 - .21 = \boxed{.79}$

Venn Diagram

$P(A \text{ only}) = .3 - .21 = .09$

$P(B \text{ only}) = .7 - .21 = .49$



$P(A \text{ only OR } B \text{ only}) = .09 + .49 = \boxed{.58}$

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Suppose a deck of cards has 40 cards and 3 aces. Draw 2 cards with replacement.

$P(\text{Both aces}) = \frac{3}{40} \cdot \frac{3}{40} = \boxed{\frac{9}{1600}}$

$P(\text{only one ace}) = 2 \cdot \frac{3}{40} \cdot \frac{37}{40} = \boxed{\frac{222}{1600}}$

$A\bar{A}$

$\bar{A}A$

$P(\text{NO Aces}) = \frac{37}{40} \cdot \frac{37}{40} = \boxed{\frac{1369}{1600}}$

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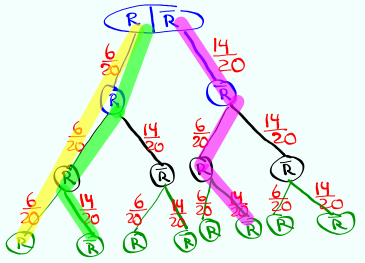
A box has 20 balls, 6 of them are red color, and we select 3 balls with replacement.



$P(RRR) = \frac{6}{20} \cdot \frac{6}{20} \cdot \frac{6}{20} = \frac{3}{10} \cdot \frac{3}{10} \cdot \frac{3}{10} = \frac{27}{1000}$

$P(RR\bar{R}) = \frac{6}{20} \cdot \frac{6}{20} \cdot \frac{14}{20} = \frac{3}{10} \cdot \frac{3}{10} \cdot \frac{7}{10} = \frac{63}{1000}$

$P(\bar{R}R\bar{R}) = \frac{14}{20} \cdot \frac{6}{20} \cdot \frac{14}{20} = \frac{7}{10} \cdot \frac{3}{10} \cdot \frac{7}{10} = \frac{147}{1000}$

P(at least 1 red) = 1 − P(No reds)

RRR, $RR\bar{R}$, $R\bar{R}R$, $R\bar{R}\bar{R}$, $\bar{R}RR$, $\bar{R}R\bar{R}$, $\bar{R}\bar{R}R$, $\bar{R}\bar{R}\bar{R}$

$= 1 - \frac{14}{20} \cdot \frac{14}{20} \cdot \frac{14}{20}$

$= 1 - \frac{7}{10} \cdot \frac{7}{10} \cdot \frac{7}{10} = \boxed{\frac{657}{1000}}$

Sample Space = A complete list of all outcomes

Jul 9-5:03 PM

There are 4 females, 6 Males. We need to select 2 people. (No replacement)

Sample Space: FF, FM, MF, MM

$P(FF) = \frac{4}{10} \cdot \frac{3}{9} = \frac{12}{90}$

$P(1F \& 1M) = 2 \cdot \frac{4}{10} \cdot \frac{6}{9} = \frac{48}{90}$

$P(MM) = \frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90}$

# Females (L1)	P(# Females) (L2)
2	$\frac{12}{90}$
1	$\frac{48}{90}$
0	$\frac{30}{90}$

STAT → CALC → 1:1-Var Stats

Menu → List: L1, Freq List: L2 → Calculate

No Menu → L1, L2 → Enter

$\bar{x} = .8$

$S =$ Blank

$n = 1$ ← Total prob.

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A piggy bank has 8 nickels & 2 quarters, we shake it to drop 2 coins.

Sample space:
- N N → 10¢ $P(10¢) = \frac{8}{10} \cdot \frac{7}{9} = \frac{56}{90}$
- N Q, Q N → 30¢ $P(30¢) = 2 \cdot \frac{8}{10} \cdot \frac{2}{9} = \frac{32}{90}$
- Q Q → 50¢ $P(50¢) = \frac{2}{10} \cdot \frac{1}{9} = \frac{2}{90}$

Total ¢	P(Total ¢)
10¢	$\frac{56}{90}$
30¢	$\frac{32}{90}$
50¢	$\frac{2}{90}$

L1 = Total ¢, L2 = P(Total ¢)

STAT ▶ CALC
1: 1-Var Stats

Using L1 & L2

$\bar{x} = 18$

S = Blank

$n = 1$ ← Total Prob.

Jul 9-5:30 PM

Multiplication Rule

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

A happens, then B happens

Given

A standard deck of playing cards has 52 cards, 12 face cards, and 4 Aces.

Draw 2 different Cards (NO replacement)

$P(2 \text{ face cards}) = \frac{12}{52} \cdot \frac{11}{51}$

$= \boxed{\frac{11}{221}}$

$P(2 \text{ Aces}) = \frac{4}{52} \cdot \frac{3}{51} = \boxed{\frac{1}{221}}$

what about draw 3 cards,

$P(3 \text{ face cards}) = \frac{12}{52} \cdot \frac{11}{51} \cdot \frac{10}{50} = \boxed{\frac{11}{1105}}$

Jul 9-5:54 PM

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

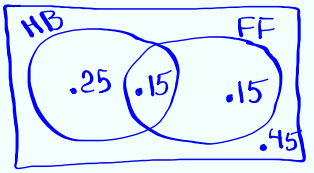
$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Conditional Prob.

$P(HB) = .4$

$P(FF) = .3$

$P(HB \text{ and } FF) = .15$



Total 1

$$P(FF \mid HB) = \frac{P(HB \text{ and } FF)}{P(HB)} = \frac{.15}{.4} = .375$$

$$P(HB \mid FF) = \frac{P(HB \text{ and } FF)}{P(FF)} = \frac{.15}{.3} = .5$$

Jul 9-6:01 PM

$P(\text{Shirt}) = .6$

$P(\text{Pants}) = .5$

$P(\text{Shirt} \mid \text{Pants}) = .8$

$$P(S|P) = \frac{P(S \text{ and } P)}{P(P)}$$

$$.8 = \frac{P(S \text{ and } P)}{.5}$$

Cross-Multiply

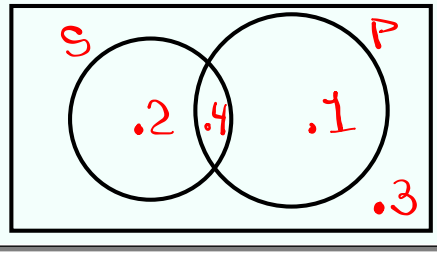
$P(S \text{ and } P) = .4$

$P(\text{Shirt and Pants})$

$$P(\text{Pants} \mid \text{Shirt}) = \frac{P(\text{Shirt \& Pants})}{P(\text{Shirt})}$$

Total 1

$$= \frac{.4}{.6} = \frac{2}{3} = .667$$



Jul 9-6:09 PM

There are 5 students and I need to select 2 of them.

Adam, Bill, Carol, David, Emily

~~AB~~ ~~AC~~ ~~AD~~ ~~AE~~

BA ~~BC~~ ~~BD~~ ~~BE~~

CA CB ~~CD~~ ~~CE~~

DA DB DC ~~DE~~

EA EB EC ED

20 ways to select 2 people. but if order does not matter

10 ways

$5C_2 = \frac{5!}{2! \cdot (5-2)!}$

$= \frac{5!}{2! \cdot 3!}$

$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$

10

Combination formula

$_nC_r = \frac{n!}{r! \cdot (n-r)!}$

n different objects

Select r of them

(order does not matter)

5 MATH ▶ PRB ↓ 3:nCr 2 Enter

10

Jul 9-6:18 PM

12 players

Need 5 to start the game

How many ways this can be done?

order does not matter.

$_{12}C_5 = 792$

CA Lotto

50 Numbers

Choose 5

in any order

How many ways can this be done?

$_{50}C_5 = 2{,}118{,}760$

Jul 9-6:28 PM

5 Females, 7 Males

Need to Select 3 different People.

1) How many ways can this be done?

$$_{12}C_3 = 220$$

2) How many ways can we Select 3 Males?

$$_7C_3 = 35$$

3) P(Selecting 3 Males) = $\frac{\text{Total \# of Male Selections}}{\text{Total \# of Selections}}$

$$= \frac{35}{220} = \frac{7}{44}$$

4) $P(3 \text{ Females}) = \frac{_5C_3}{_{12}C_3} = \frac{10}{220} = \frac{1}{22}$

5) $P(2 \text{ Males} \text{ & } 1 \text{ Female}) = \frac{_7C_2 \cdot {}_5C_1}{_{12}C_3} = \frac{105}{220} = \frac{21}{44}$

Jul 9-6:34 PM

Standard deck of playing cards has 52 cards, 12 face cards, 4 Aces. Draw 5 cards, No replacement, order does not matter.

$P(3 \text{ face cards} \text{ & } 2 \text{ Aces}) = \frac{_{12}C_3 \cdot {}_4C_2}{_{52}C_5}$

$= \frac{1320}{2\,598\,960} \approx \boxed{5.1 \times 10^{-4}}$

$P(2 \text{ face cards} \text{ & } 3 \text{ Aces}) = \frac{_{12}C_2 \cdot {}_4C_3}{_{52}C_5}$

$= \frac{264}{2598960}$

$\approx 1.02 \times 10^{-4}$

$P(2 \text{ face cards} \text{ & } 2 \text{ Aces})$

$= \frac{_{12}C_2 \cdot {}_4C_2 \cdot {}_{36}C_1}{_{52}C_5} = \frac{14256}{2598960}$

$= \boxed{.005}$

Jul 9-6:45 PM

4 Females, 6 Males were hired.

5 Morning, 3 afternoon, 2 graveyard.

P(at least 1 Female for afternoon shift)

$= 1 - P(\text{All Males})$

$= 1 - \frac{{}_6C_3}{{}_{10}C_3} = 1 - \frac{1}{6} = \frac{5}{6}$

P(at least 1 Male in the morning shift) = 1

P(at least 1 Male in the graveyard Shift)

MM
MF
FM
FF

$= 1 - P(\text{All Females})$

$= 1 - \frac{{}_4C_2}{{}_{10}C_2} = \boxed{\frac{13}{15}}$

SG 12 & 13

Jul 9-6:57 PM